Section 8.2

Arc Length and Surface Area

Get Ready to Vote!

Arc Length and Surface Area (8.2)

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Multiple Choice
A B C D E

Which of the following is the correct expression for the **arc length** of the curve $y = x^{3/2}$ from $x = 0$ to $x = 4$?

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(A) $\displaystyle\int_0^4 x^{3/2}\,dx$

(B) $\displaystyle\int_0^4 |x|^{3/2}\,dx$

(C) $\displaystyle\int_0^4 \sqrt{1 + \tfrac{9}{4}x}\,dx$

(D) $\displaystyle\int_0^4 \sqrt{1 + x^{3/2}}\,dx$

(E) $\displaystyle\int_0^4 \sqrt{1 + \tfrac{3}{2}x^{1/2}}\,dx$

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Answer: (C) . Note that $(y')^2 = \left(\tfrac{3}{2}x^{1/2}\right)^2 = \tfrac{9}{4}x$.

Arc Length and Surface Area (8.2)

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The curve $y = \frac{1}{2}x - 1$, for $2 \leq x \leq 5$, is revolved around the **x-axis**. What surface do we get?

Arc Length and Surface Area (8.2)

Multiple Choice
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The curve $y = \frac{1}{2}x - 1$, for $2 \le x \le 5$, is revolved around the **x-axis**. What surface do we get?

(A) A cylinder.

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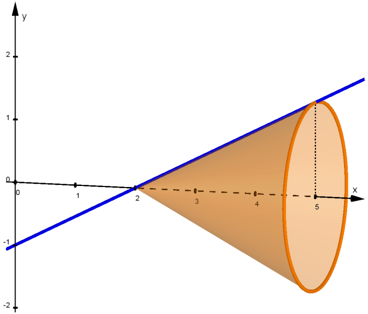

Answer: **(B)** .

Arc Length and Surface Area (8.2)

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Multiple Choice
A B C D E

Which integral gives the **surface area** of the solid obtained by revolving the curve $y = x^2$, for $-1 \le x \le 1$, about the **x-axis**?

Arc Length and Surface Area (8.2)

Multiple Choice
A B C D E

Which integral gives the **surface area** of the solid obtained by revolving the curve $y = x^2$, for $-1 \le x \le 1$, about the **x-axis**?

(A) $2\pi \int_{-1}^{1} x\sqrt{1+4x^2}\,dx$

(B) $2\pi \int_{-1}^{1} x^2\sqrt{1+x^2}\,dx$

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(C) $\int_{-1}^{1} \sqrt{1+4x^2}\,dx$

Answer: (E) . We use the formula $S = 2\pi \int_{a}^{b} f(x)\sqrt{1+[f'(x)]^2}\,dx$.

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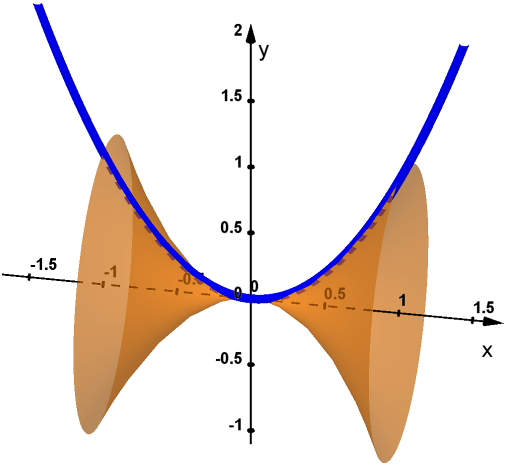

Arc Length and Surface Area (8.2)

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Multiple Choice
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Suppose that $f(x)$ is a function with domain $[0,6]$,

and that $g(x) = \frac{1}{2}f(2x)$ (with domain $[0,3]$) . What is the relation

between the length L_f of $y = f(x)$ and the length L_g of $y = g(x)$?

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(A) $L_f = \frac{1}{2} L_g$

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(C) $L_f = 2 \cdot L_g$

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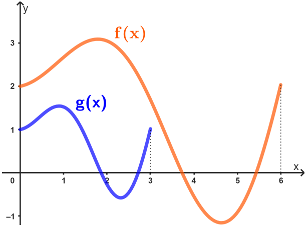

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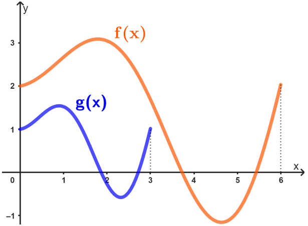

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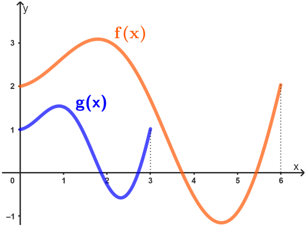

Answer: (C), since $g'(x) = f'(2x)$ and so $L_f = \int_0^6 \sqrt{1+f'(x)^2}\,dx =$

$$= \int_0^3 \sqrt{1+f'(2u)^2} \cdot 2\,du = 2 \cdot L_g$$ (we used the substitution $x = 2u$).

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Highlights:

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- The **arc length** of the curve $\mathbf{y=f(x)}$, for $\mathbf{a \le x \le b}$, is given by $\mathbf{s} = \int_{\mathbf{a}}^{\mathbf{b}} \sqrt{\mathbf{1+[f'(x)]^2}}\,\mathbf{dx}$.

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- This formula is obtained by **approximating** the curve with **line segments**, and then taking a limit.

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- The surface generated by a revolving graph can be **approximated** by slanted bands.

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- The **arc length** of the curve $y = f(x)$, for $a \le x \le b$, is given by $s = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$.

- This formula is obtained by **approximating** the curve with **line segments**, and then taking a limit. (The **MVT** is used.)

- The surface generated by a revolving graph can be **approximated** by slanted bands.

- By taking a limit, we obtain **the exact formula** (for $f(x) \ge 0$):

$$S = 2\pi \int_a^b f(x) \cdot \sqrt{1 + [f'(x)]^2}\, dx$$

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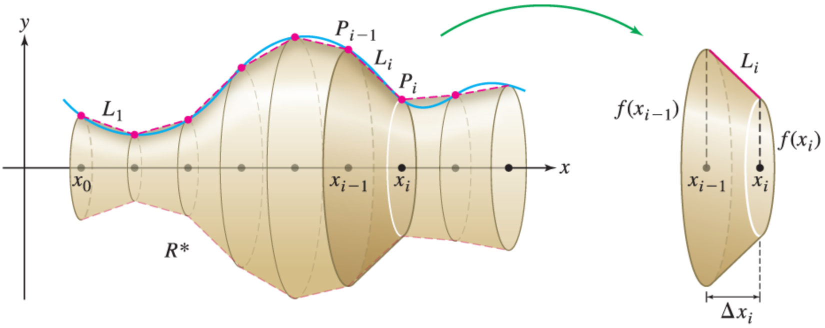

Arc Length and Surface Area (8.2)

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Compute the **arc length** of the following curve(s).

(a) $y = \frac{x^2}{2} - \frac{\ln x}{4} + 3$ for $1 \leq x \leq 3$.

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Compute the **arc length** of the following curve(s).

(a) $y = \frac{x^2}{2} - \frac{\ln x}{4} + 3$ for $1 \leq x \leq 3$.

(b) $y = \frac{e^{-x} + e^{x}}{2}$ for $-1 \leq x \leq 1$.

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Confirming the formula for the surface area of a cone.

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Confirming the formula for the surface area of a cone.

A cone of base radius $\mathbf{r}$ and height $\mathbf{h}$ can be obtained by revolving the line $\mathbf{y} = \frac{\mathbf{r}}{\mathbf{h}} \cdot \mathbf{x}$, for $\mathbf{0} \leq \mathbf{x} \leq \mathbf{h}$, around the **x-axis**.

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(a) **Draw a diagram** that represents the given information.

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Confirming the formula for the surface area of a cone.

A cone of base radius $\mathbf{r}$ and height $\mathbf{h}$ can be obtained by revolving the line $\mathbf{y = \frac{r}{h} \cdot x}$, for $\mathbf{0 \le x \le h}$, around the **x-axis**.

(a) **Draw a diagram** that represents the given information.

(b) Use the formula $\mathbf{S = 2\pi \int_a^b f(x)\sqrt{1 + [f'(x)]^2}\, dx}$ to compute **the surface area** of the cone (without the base).

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(b) Use the formula $\mathbf{S} = 2\pi \int_a^b \mathbf{f(x)} \sqrt{1 + [\mathbf{f}'(\mathbf{x})]^2}\, \mathbf{dx}$ to compute **the surface area** of the cone (without the base).

You should get: $\pi \mathbf{r} \sqrt{\mathbf{r}^2 + \mathbf{h}^2}$.